Interpreting resistivity from lightning strikes

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SUMMARY

Data routinely recorded for lightning strikes may be adequate for measuring subsurface resistivity.

INTRODUCTION

Lightning has been studied for many years as a purely meteorological phenomenon. Recent examples include Bruning et al. (2010), Gatlin and Goodman (2010), Zhang et al. (2012), and many others. In these studies the earth has been viewed as a uniform sink for electrostatic discharges.

But as geoscientists we know that the earth is not uniform, and in particular that its electrical properties are not uniform. Electrical resistivity variations are the most fundamental property measured in well logging, and these resistivity variations are used (along with other measurements) to identify lithology, porosity, and pore fluid composition. Surely the same variations in resistivity will affect lightning strike characteristics and location. To put it into perspective, a single typical lightning stroke dissipates energy of 100-150 kWh – three to eight days electricity consumption for the average U.S. household – with peak currents measured in kiloamperes.

This paper considers one way of separating the effects on lightning of geological variations from the effects of atmospheric (or meteorological) effects.

Data

Available lightning data records from traditional ground-based detection networks such as the National Lightning Detection Network (NLDN) contain (for each cloud-to-ground lightning stroke) precise time of the stroke, the geographic location, the peak current, the rise time (the time from onset to peak current), the peak-to-zero time, and a number of measures of the accuracy of location and of the other measurements. Published accounts of the accuracy of these measurements include Chen et al. (2012) and Murphy et al. (2008).

Available lightning data records from VLF detection networks do not include rise time and peak-to-zero time.

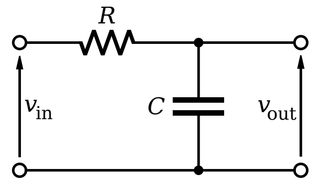

Figure 1: RC circuit

THEORY

A lightning strike can be considered as an RC (resistance-capacitor) circuit (Figure 1).

V_{out} is the lightning stroke, and V_{in} is the static charge accumulating in the thunderstorm. This is the circuit of a relaxation oscillator first described by Pearson and Anson (1921), with the power supply at V_{in} and the neon tube at V_{out}. Conceptually, the discharge of the neon tube is the equivalent of lightning. In the same way as the capacitor recharges once the current becomes too small to keep it active, the charge in a thunderstorm will build up again once the current in a lightning stroke drops below the level needed to maintain ionization of the atmosphere. A better approximation to lightning is shown in Figure 2, where the bottom part of the diagram includes resistance (of the earth) and the top part (the charged cloud) has no resistance. But this circuit would give an instantaneous discharge, as the closed circuit with the capacitor and the lightning has no resistive component.

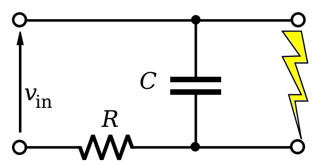

Figure 2: A better approximation of the lightning circuit

The significant resistance is the earth resistance, which is the average of the resistance between the strike point and all the points on the ground surface making up the lower plate of the capacitor. Note that the this is a parallel circuit average:

$$R_{0-n} = \frac{1}{\frac{1}{R_0} + \frac{1}{R_1} + \cdots + \frac{1}{R_n}} \tag{1}$$

Figure 3 shows a circuit closer to the real world of lightning. This circuit takes into account the resistance limiting the speed of the recharge (R_1), and the resistance

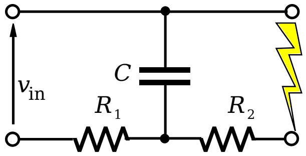

Figure 3: An even better approximation of the lightning circuit

limiting the speed of the discharge (R_2). The one component missing is inductance, which must be present because lightning produces an electromagnetic wave. But it also must be small: Uman (1984), p.216, estimates radiated electromagnetic power to be about 1% of the total power in a return stroke. For lightning, the inductance will mainly affect the rise time. Once the peak is reached, current will decay exponentially. Given the analogy of the lightning system to an electrical circuit, we can correlate the circuit components to actual physical objects. What are the physical relationships between these electrical components?

- C is the capacitor formed by the atmospheric static charge (usually at or near the bottom of the cloud for negative strikes, and near the top of the cloud for positive strikes) as one plate, the atmosphere as the dielectric, and the conducting ground surface as the other plate.
- R_1 is the resistance of the earth below the thundercloud as it affects the build-up of static charge within the thundercloud.
- R_2 is the resistance of the earth below the cloud as it affects the movement of electrons to re-balance static charges after the lightning completes the circuit.

The voltage across the earth resistance as a function of time t from the peak current is given by:

$$V_t = V_0 e^{-t/RC} \tag{2}$$

where V_0 is the voltage at the instant of the lightning initiation. The dielectric strength of air is about 3.0 MV/m ($V_0 = 3.0d$ for megavolts or $V_0 = 3000000d$ for volts), so if a cloud base is at a typical level of 500 m the value of V_0 is about 1500 MV for a negative strike. This is probably an overestimate: decrease in pressure with altitude reduces dielectric strength, and the presence of water droplets probably does also.

From the relationship of voltage, current and resistance ($V = IR$ hence $R = \frac{V}{I}$) we can calculate $R = \frac{1{,}500{,}000{,}000}{100{,}000}$ or 15 kΩ for a typical peak current of 100 kA.

By using the same substitution in the the decay function:

$$I_t R = I_0 R e^{-t/RC} \tag{3}$$

or

$$I_t = I_0 e^{-t/RC} \tag{4}$$

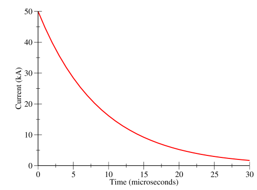

Figure 4: Decay of current from Equation 4.

Figure 4 shows values for this equation assuming peak current of 50.0 kA, an effective capacitor area of 1.0 km^2, and effective earth resistance of 1000Ω, and a cloud-to-ground distance of 1000 m.

As lightning measurements over an extended period give multiple measurements at the same location, it should be possible to calculate values of R for each location. The value of C could be expected to be mainly meteorological, but the value of R should be almost entirely geological.

Taking the natural logarithm of both sides:

$$ln(I_t) = ln(I_0) - t/RC \tag{5}$$

$$t/RC = ln(I_0) - ln(I_t) \tag{6}$$

$$RC = \frac{t}{ln(I_0) - ln(I_t)} \tag{7}$$

$$R = \frac{t}{C(ln(I_0) - ln(I_t))} \tag{8}$$

Isolating R is not trivial. Figure 5 shows the direction of current flow from a point electrode with uniform resistivity below the surface (Loke, 2012, Figure 1.1). A lightning strike is an analog to a point electrode. The measured resistance (R_2 in Figure 3) is heavily weighted towards the rocks close to the lightning strike because the effective cross-section of the conducting material increases in proportion to the square of the distance from the strike. One approach which might work is to assume that low peak current indicates a low cloud base, giving a thinner and more easily ruptured dielectric layer. This would also give a higher value of C. Capacitance

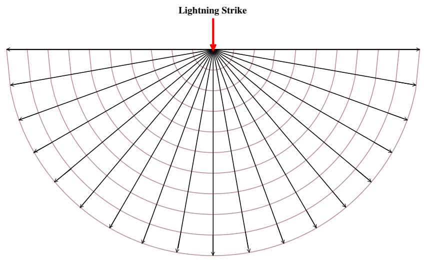

Figure 5: Equipotential surfaces (in brown) and direction of current flow from a lightning strike [after (Loke, 2012, Figure 1.1)]

is inversely proportional to the distance between the two plates of a parallel-plate capacitor, so in order to approximate capacitance we could assume an arbitrary area, and an arbitrary cloud base for an average lightning strike. We can then assume a linear relationship between peak current and cloud base height to give a value of C for each strike within a local area, thus allowing the calculation of a value of R for each strike. The mean of these values should be a robust estimate of the apparent resistance encountered by the lightning charge passing through the subsurface.

Here is how this approach might be used. The capacitance formed by the charged layer in the cloud (one plate), the atmosphere (the dielectric), and the earth (the other plate) is approximately given by:

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} \tag{9}$$

where

C is capacitance in farads;

ε_r is the relative permittivity of air (1.00);

ε_0 is the electric constant ($\varepsilon_0 \approx 8.854 \times 10^{-12} Fm^{-1}$);

A is the area affected;

d is the distance between the cloud base and the ground.

At the beginning of the stroke, we can say that the resistance (i.e. at the peak current) is given by

$$R = \frac{V_0}{I_0} \tag{10}$$

where

R is the effective ground resistance and

V_0 is the breakdown voltage of the capacitor.

Also,

$$V_0 = 3000000d \tag{11}$$

so

$$R = \frac{3000000d}{I_0} \tag{12}$$

From above, we have

$$R = \frac{t}{C(ln(I_0) - ln(I_t))} \tag{13}$$

and from 9 assuming $\varepsilon_r = 1.0$

$$C = \varepsilon_0 \frac{A}{d} \tag{14}$$

substituting for C

$$R = \frac{td}{\varepsilon_0 A(ln(I_0) - ln(I_t))} \tag{15}$$

The effective area of the capacitor discharged by the lightning might be assumed to be a function of the cloud height, d. A possible approximation is $A = \pi d^2$ (the area of a circle with radius d). Substituting in the equation for R,

$$R = \frac{td}{\varepsilon_0 \pi d^2(ln(I_0) - ln(I_t))} \tag{16}$$

or

$$R = \frac{t}{\varepsilon_0 \pi d(ln(I_0) - ln(I_t))} \tag{17}$$

Substituting from equation 12

$$\frac{3000000d}{I_0} = \frac{t}{\varepsilon_0 \pi d(ln(I_0) - ln(I_t))} \tag{18}$$

or

$$d^2 = \frac{tI_0}{3000000\varepsilon_0 \pi(ln(I_0) - ln(I_t))} \tag{19}$$

We now have a solution for d, the distance from the cloud base (or the center of the charge) to the ground. From equation 12 we can then calculate the electrical resistance encountered by the discharging capacitor. Assuming the ground resistivity distribution remains the same, and most of the resistance is in the earth, not in the atmosphere, each lightning strike at the same location encounters the same resistance, but the distance from the cloud base to the ground will vary. The inputs to these equations are:

- I_0 (the peak current)
- t (the peak to zero time)
- I_t (the current at "zero" current)

Only the last item presents problems: the actual "peak-to-zero" time would theoretically be infinite. The measured time is the time at which the signal disappears into the background noise. What is the current corresponding to this signal? We have to make an assumption here: half the smallest peak current would be a good starting place. Values for resistivity rather than just for resistance would be more useful. If we assume the depth of penetration of the electrical current is proportional to the distance d, a crude estimate can be made of the actual resistivity.

EXAMPLE

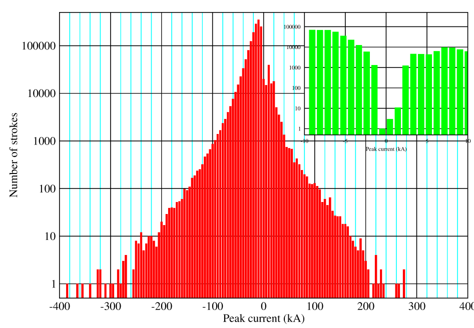

Figure 6: Peak current distribution for 1582072 lightning strikes in an area of about 6600 km^2 in Harris County and adjoining areas between 2000 and 2011. The inset plot shows detail for peak current from -10 to +10 kA.

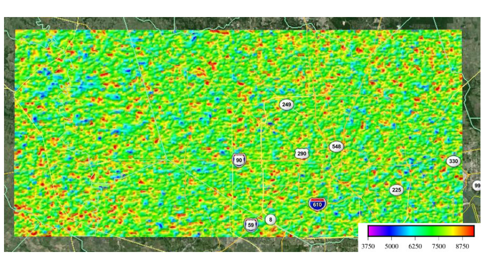

Figure 7: Calculated resistivity values (Ωm) for area including most of Harris County, Texas, from west of Sealy to Baytown, and from Sugar Land to Tomball.

In reality, an abrupt cutoff of the current from the lightning will occur when the current is no longer large enough to maintain ionization over the path of the lightning stroke. Figure 6 suggests that in the test area the cutoff is about 1.0 kA. Figures 7, 8 and 9 show surface resistivity generated from twelve years of archived lightning data over a major metropolitan area.

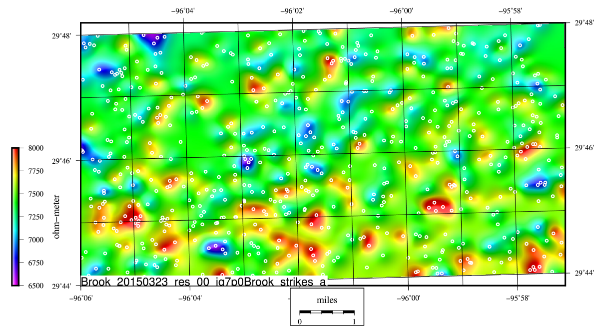

Figure 8: An area around the intersection of Interstate 10 and the Brazos River, plotting the same data as Figure 7. Lightning strikes are shown as white circles.

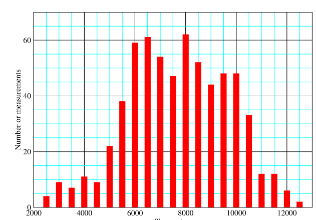

Figure 9: Histogram of values mapped in Figure 8

CONCLUSION

An estimate of the resistivity of near surface formations can be made from recorded lightning events. Because a depth of penetration is associated with each resistivity measurement, this potentially allows calculation of resistivity as a function of depth at any location with multiple lightning strikes.

REFERENCES

Bruning, E. C., W. D. Rust, D. R. MacGorman, M. I. Biggerstaff, and T. J. Schuur, 2010, Formation of charge structures in a supercell: Monthly Weather Review, **138**, 3740–3761. (doi: http://dx.doi.org/10.1175/2010MWR3160.1).

Chen, L., Y. Zhang, W. Lu, D. Zheng, Y. Zhang, S. Chen, and Z. Huang, 2012, Performance evaluation for a lightning location system based on observations of artificially triggered lightning and natural lightning flashes: Journal of Atmospheric and Oceanic Technology, **29**, 1835–1844. (doi: http://dx.doi.org/10.1175/JTECH-D-12-00028.1).

Gatlin, P. N., and S. J. Goodman, 2010, A total lightning trending algorithm to identify severe thunderstorms: Journal of Atmospheric and Oceanic Technology, **27**, 3–22. (doi: http://dx.doi.org/10.1175/2009JTECHA1286.1).

Loke, M. H., 2012, Tutorial: 2-d and 3-d electrical imaging surveys: online. (`http://www.geoelectrical.com/coursenotes.zip`).

Murphy, M. J., R. L. Holle, and N. W. S. Demetriades, 2008, Cloud-to-ground lightning warnings using electric field mill and lightning observations. (`http://www.vaisala.com/Vaisala%20Documents/Scientific%20papers/Cloud-to-ground_lightning_warnings_using_electric_field_mill_and_lightning_observations.pdf`).

Pearson, S. O., and H. S. G. Anson, 1921, Demonstration of some electrical properties of neon-filled lamps: Proceedings of the Physical Society of London, **34**, 175–176. (doi: 10.1088/1478-7814/34/1/435).

Uman, M. A., 1984, Lightning: Dover.

Zhang, W., Y. Zhang, D. Zheng, and X. Zhou, 2012, Lightning distribution and eyewall outbreaks in tropical cyclones during landfall: Monthly Weather Review, 3573–3586. (doi: http://dx.doi.org/10.1175/MWR-D-11-00347.1).